Active Sensing and Control

Lecture Notes for the 2006 IROS Workshop

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Outline

- Observability of nonlinear control systems
- Active sensing and control for nonlinear systems
- Active exploration
- Active sensing for group consensus
- Closing remarks

Nonlinear Observability

Consider a nonlinear control system

$$\begin{aligned} \dot{x} &= f(x) + \sum_{i=1}^{m} g_i(x) u_i \\ y &= h(x). \end{aligned}$$

When

$$f(x) = Ax, \quad [g_1(x) \cdots g_m(x)] = B, \quad h(x) = Cx,$$

We have a linear system.

Nonlinear Observability

Roughly speaking, observability implies that if $x_1 \neq x_2$, then

$$y(t, x_1, u) \neq y(t, x_2, u), \; t \in [0, T],$$

for some admissible u.

- For linear systems, observability is decided by (C,A) only.
- For nonlinear systems, observability depends on h(x), f(x) as well as g(x).

→ Uniform and non-uniform observability

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Example: Attitude estimation

$$
\begin{aligned}
\dot{x}_1 &= -u_3 x_2 + u_1 \\
\dot{x}_2 &= u_3 x_1 + u_2 \\
\dot{x}_3 &= \tau x_1 - \tau x_3 \\
y &= x_3
\end{aligned}
$$

This is the bilinear model for using a low-pass sensor to measure the pitch and roll angles of a rigid body.

One can see easily that in order to have observability, the open-loop control u_3 needs to satisfy certain constraint.

Nonlinear Observer

In general, an observer should take the following form:

$$\dot{\hat{x}} = p(\hat{x}, h(x(t)), u(t))$$

with the corresponding error dynamics asymptotically stable. Consequently, $\|x(t) - \hat{x}(t)\| \to 0$ as $t \to \infty$.

Active sensing → find optimal u(t)

Observability and Observer

- For linear systems, observability implies the existence of observers.
- For linear systems, observability is only **necessary** for the existence of observers.

Example. Consider

$$\begin{aligned} \dot{x}_1 &= -x_1 + x_2^3 \\ \dot{x}_2 &= x_2 + x_1^2 \\ y &= x_1. \end{aligned}$$

One can easily show that this system is observable. However, no Observer exists for this system.

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Separation Principle

- For linear systems, one can first design an optimal control u(x) assuming all the states x are available;
- Then design an observer x(t) to estimation x(t);
- Finally, use u(x(t)) in implementation.

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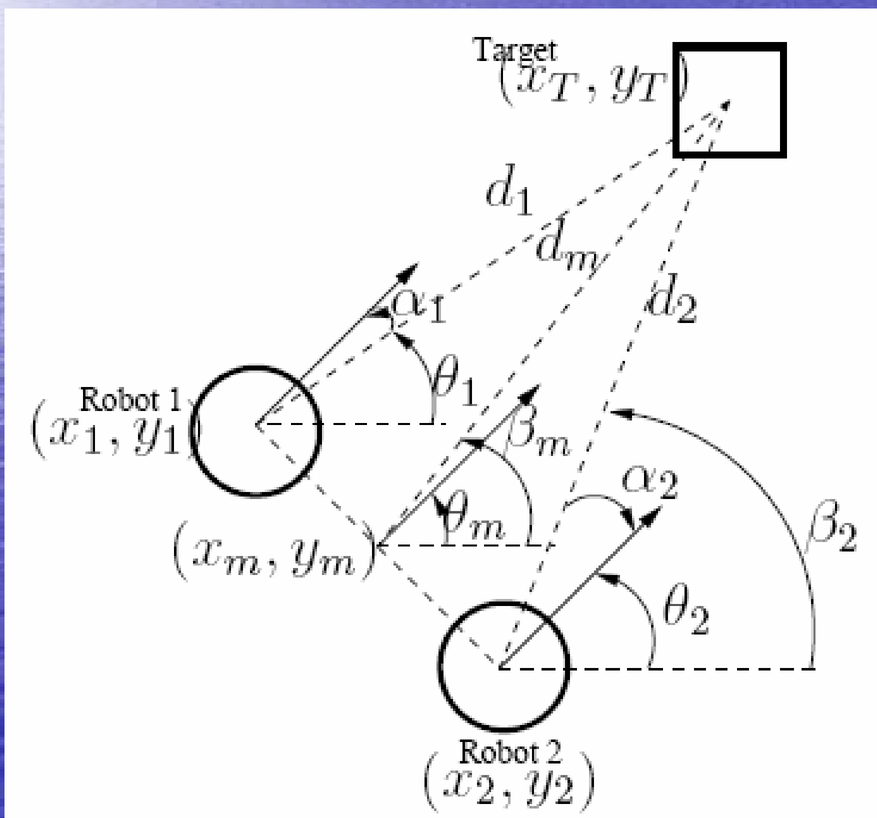
For nonlinear systems, the separation principle does not hold in general.

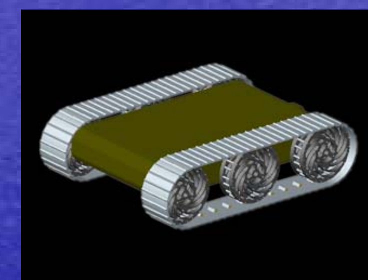
We have to treat sensing and control in an integrated fashion.

Active sensing and control

Integration of Sensing and Control

Consider several robots equipped with directional and range sensors tracking a moving target:





Active Sensing and Control

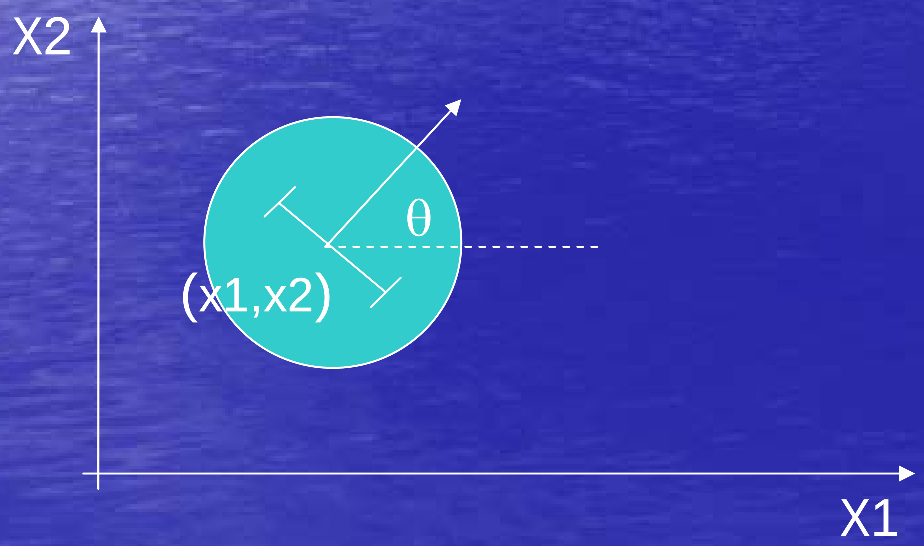
In mobile robotic systems, one typically uses sensors that interact with the environment, such as lasers and video. Based on this background, we consider the following system:

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x, s) \\ x_e(s) &= \phi(s), \end{aligned}$$

where $x \in \mathbb{R}^n$, $y \in \mathbb{R}^p$, $u \in \mathbb{R}^m$ and $s \in \mathbb{R}^q$. $x_e(s) = \phi(s)$ defines manifolds in $\mathbb{R}^n$ that model the environment.

Case Study

We consider an oriented robot in the plane, using the so-called unicycle model. Namely, the state of the system is described by $((x_1, x_2), \theta) \in \mathbb{R}^2 \times S^1$ and it has two control inputs, the translational velocity v and the angular velocity ω.



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Unicycle

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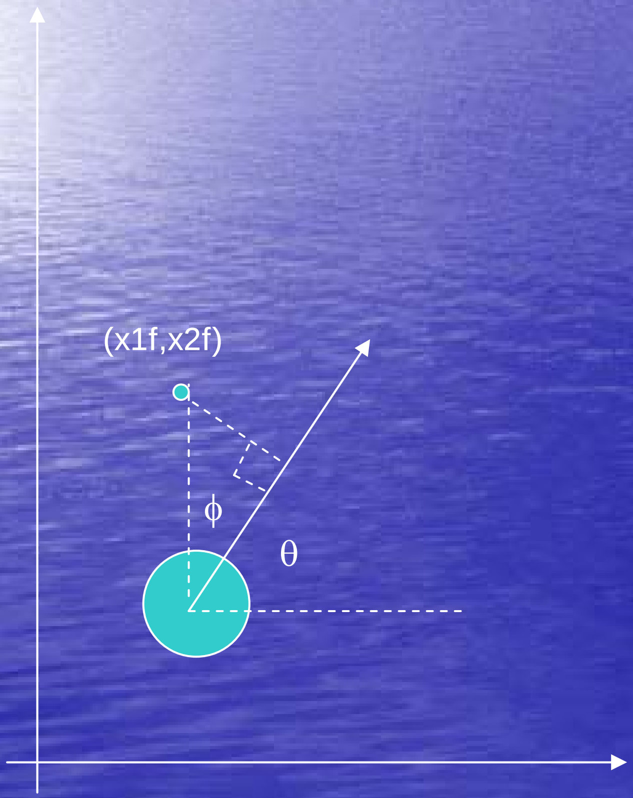
The governing dynamics are

$$\begin{aligned} \dot{x}_1 &= v\cos\theta \\ \dot{x}_2 &= v\sin\theta \\ \dot{\theta} &= \omega. \end{aligned}$$

Furthermore, we assume that the robot is equipped with video camera and/or range-measuring sensors.

Sensor: Camera

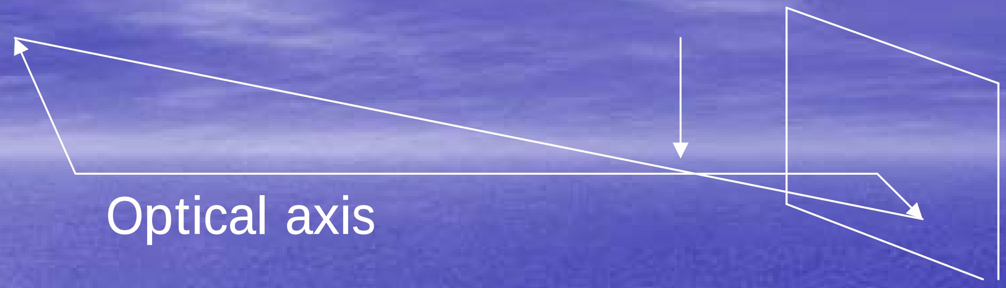


We suppose the altitude of the robot does change and a video camera with focal length one is mounted on top of the robot. Given a feature point (x_{1f}, x_{2f}), we then have the image of the point as:

$$y = \tan(\phi),$$

where $\phi = arctan(x_{2f} - x_2, x_{1f} - x_1)$.

Camera in 3D



We suppose the point's position is (x_1, x_2, x_3) in 3D. Then in the image plane we have $(y_1, y_2) = (f\frac{x_1}{x_3}, f\frac{x_2}{x_3})$. The relative motion of the point $(f = 1)$:

$$\begin{pmatrix} \dot{x}_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} \frac{x_1}{x_3} \\ \frac{x_2}{x_3} \end{pmatrix} \qquad x_3 \neq 0,$$

Range Sensors

Case Study One

We consider mobile robots equipped with video cameras. Known point features in the environment are used for localization.

Consider

$$\begin{aligned} \dot{x} &= A(t)x + Bu(t) \\ y &= \frac{Px}{q^T x}, \end{aligned}$$

where $A(\cdot)$ is a C^1 $n \times n$ matrix and both $\|A(t)\|$ and $\|\dot{A}(t)\|$ are uniformly bounded, P is a $m \times n$ matrix and q is $n \times 1$ matrix.

Rewrite the output equation as

$$(P - yq^T)x = 0.$$

This can be interpreted as if the output is defined implicitly. Using the equation above we consider the following class of observers

$$\dot{\hat{x}} = A(t)\hat{x} + Bu(t) + L(t)(P - y(t)q^T)\hat{x}.$$

Lemma. Suppose $A(\cdot)$ is uniformly Liapunov stable and let $\Phi(\tau, t)$ be the state transition matrix. Denote

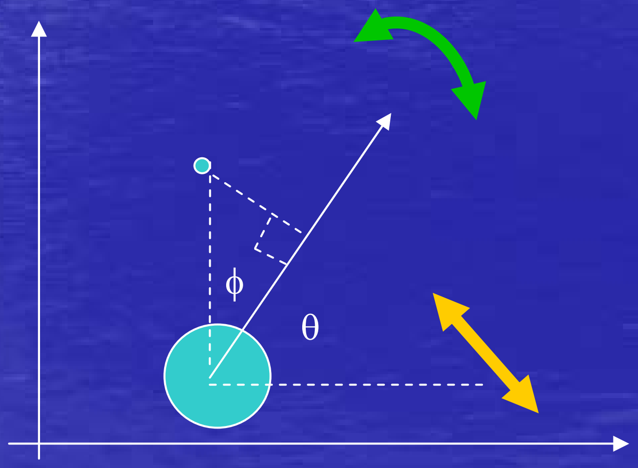
$$M(\tau) := (P - y(\tau)q^T)^T(P - y(\tau)q^T).$$

If

$$\mathcal{O} := \int_t^{t+T} \Phi^T(\tau, t)M(\tau)\Phi(\tau, t)d\tau \geq \epsilon I$$

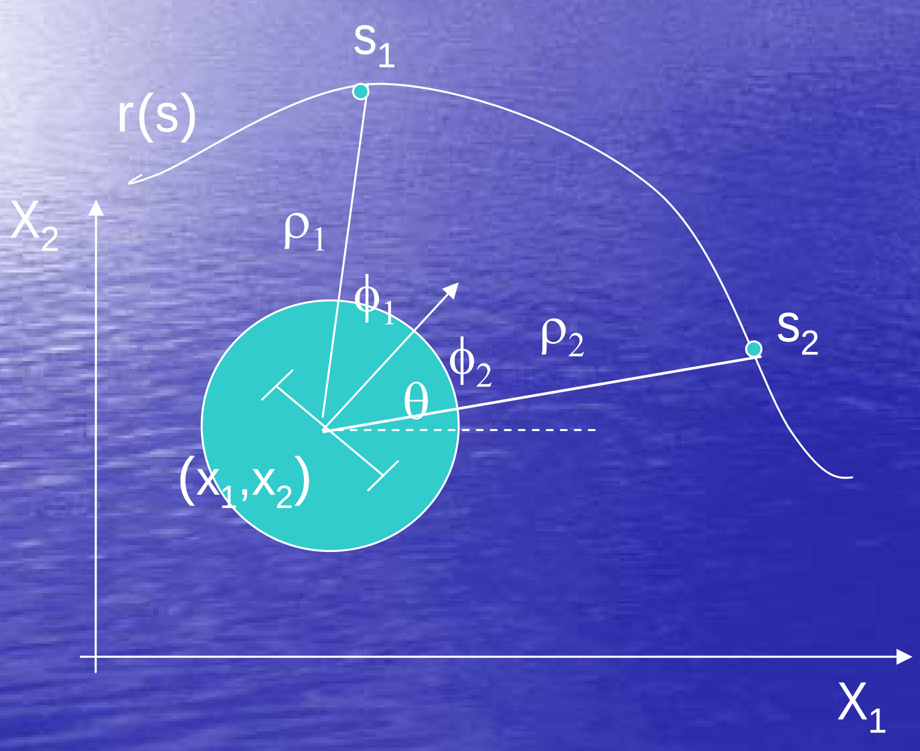
for all $t \geq t_0$, $|x_0| \leq m$ and some positive T and ϵ. Then there is an exponential observer, i.e. the error dynamics converges to zero exponentially for all $|x_0| \leq m$.

- It has been shown theoretically how we can construct such an exciting control so that the state can be observed.
- In the camera example, it is well known that quick translational movements in planes parallel to the perception plane provide very rich depth information.



Case Study Two

We consider a mobile robot with range sensors.



$$\dot{x}_1 = v\cos\theta$$
$$\dot{x}_2 = v\sin\theta$$
$$\dot{\theta} = \omega$$
$$y_1 = \rho_1$$
$$y_2 = \rho_2.$$

We suppose $r(s)$ is closed and known.

Our problem is to construct an observer for the full state of the robot. It is easy to see that this problem is equivalent to the reconstruction of orientation θ and the two parameter values $s_1, s_2 \in S^1$ corresponding to the points on the curve measured against.

In order to do this we define a (new) state variable $p = (s_1, s_2, \theta) \in S$, which we call the *parameter configuration*. Since we will be concerned with local properties only, we can consider p as an element of $\mathbb{R}^3$.

Statically Unobservable Submanifold

Using only the values of ρ_1 and ρ_2 it is easy to see that we will, in general, only be able to determine the configuration of the robot up to some curve in $\mathbb{R}^2 \times S^1$. One can picture this by following a planar curve with the tips of two extended fingers. We call this curve the *statically unobservable submanifold*. This fact in particular suggests that the nonlinear system is not always observable.

Active Relocalization and Nonlinear Observers

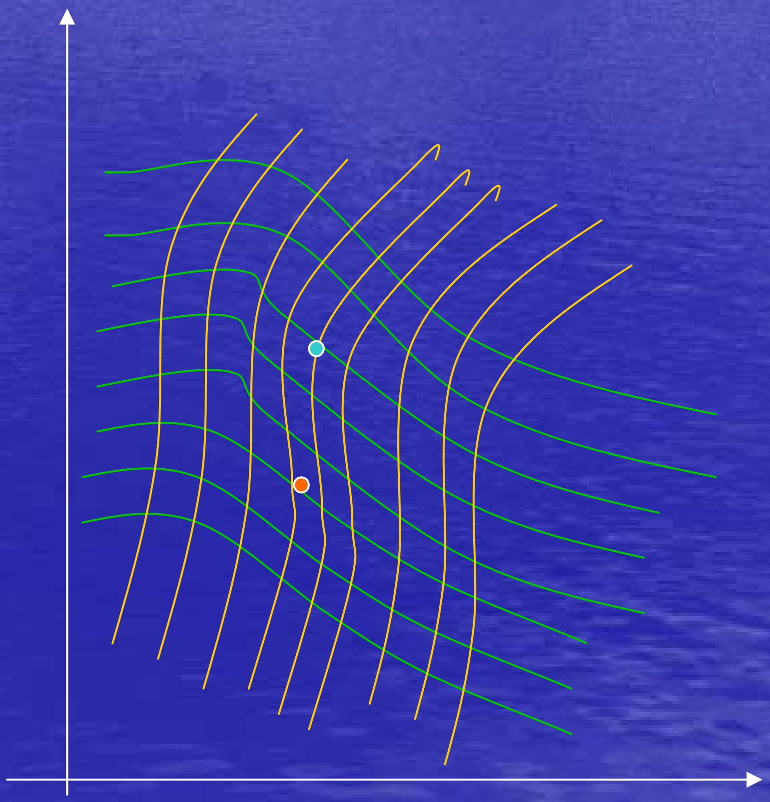
Next, we will discuss what can be done in the statically unobservable direction.

Assuming that we have obtained a point on the curve of feasible configurations by some optimization method, we would now like to localize the actual configuration of the robot by finding a point on the curve where the estimated output flow (time derivatives of the measured distances) coincide with the actual output flow.

Dynamically Unobservable Submanifold

When taking $\dot{\rho}$ into account, we find a similarly unobservable submanifold, also one–dimensional.

The theoretical basis for active non-linear observer is that under certain conditions the two different sub-manifolds can be made not parallel at the actual parameter configuration of the robot, and hence the system would be locally observable.



Active Control

The exciting control v, ω *should be designed such that* $v \sin \varphi_i - \omega \rho_i \neq 0$, *for* $i = 1, 2$.

Having one of the components zero corresponds to moving in such a way that $\dot{s}_i = 0$ for that sensor. If the sensors are directed to different sides of the steering direction and the control input is not zero, this cannot happen since ρ_1 and ρ_2 both are positive.

Observer

Under the suitable assumptions, we propose the following observer

$$\frac{d\hat{p}}{dt} = -k(\partial_{\hat{p}} z)^T z(\hat{p}) - k(\partial_{\hat{p}} F)^T F(\hat{p}),$$

where z, F define the statistically and dynamically unobservable submanifolds, $k > 0$ is a suitably tuned feedback gain. The estimation error is bounded locally and the bound can be made arbitrarily small by tuning k, provided, for example, periodic control is used.

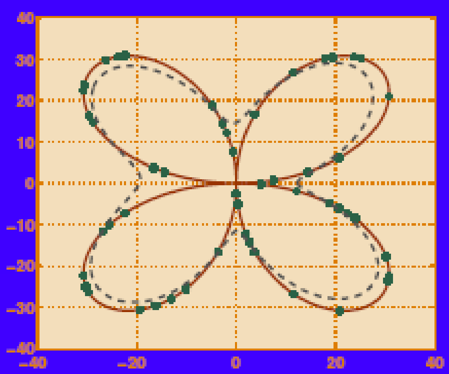
Active Exploration

- Aim:

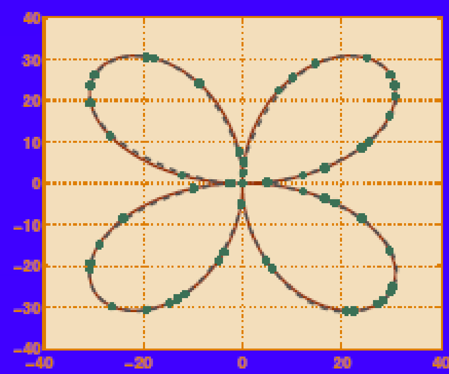
Improve the exploration strategy based on known data on the environment.

- Setup:

A vehicle servoes around an unknown object, making noise contaminated distance measurements. One vehicle makes several revolutions. On each revolution, the exploring path is modified based on the data already collected.



First revolution



12:th revolution

Smoothing Spline

Initially, the smoothing spline is found through minimizing a cost function:

$$\min_{u,r_0} J = \delta^2 \left.\frac{\partial r}{\partial \theta}\right|_{\theta=0} + \frac{1}{2}\int_0^T u(\theta)^2 d\theta + \varepsilon^2 \sum_{i=1}^{N} (\theta_i - \theta_{i-1})(z_i - r(\theta_i))^2$$

$$\text{subject to } \begin{aligned} \frac{\partial^2 r}{\partial \theta^2} &= u \\ r_0 &= r(0) = r(T). \end{aligned}$$

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Recursive Path Planning: Making Use of Previous Estimations

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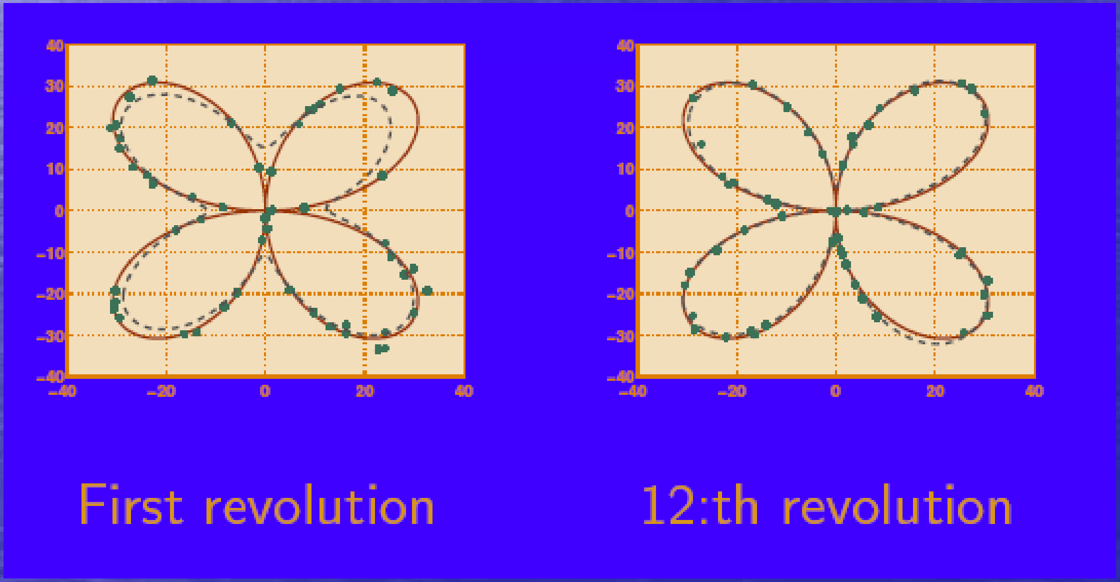
- Adding more data $\Rightarrow$ the spline converges towards the real curve Drawback: the problem grows.
- Recursive method: the spline x^{k-1} is included in cost function for spline x^k:

$$\min_{u^k, x_0^k} J_k = \frac{1}{2}(x_0^k - x_0^{k-1})^T P_0^{-1}(x_0^k - x_0^{k-1}) + \frac{1}{2}\int_0^T (u^k(t) - u^{k-1}(t))^T Q^{-1}(u^k(t) - u^{k-1}(t))dt + \sum_{i=1}^{N}(t_i - t_{i-1})(z_i^k - Cx^k(t_i))^T R_0^{-1}(z_i^k - Cx^k(t_i))$$

$$\dot{x}^k = Ax^k + Bu^k, \qquad x_0^k = x^k(0) = x^k(T).$$

Exploration

- Local path planning using splines.
- Virtual vehicle approach for path following.



10% measurement error

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Consensus Problem

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- Now we consider a system of N agents:

$$\dot{x}_i = u_i, \; i = 1, \cdots, N$$

where x_i can be viewed as heading, position or other quantities.

- We define the consensus problem as follows:

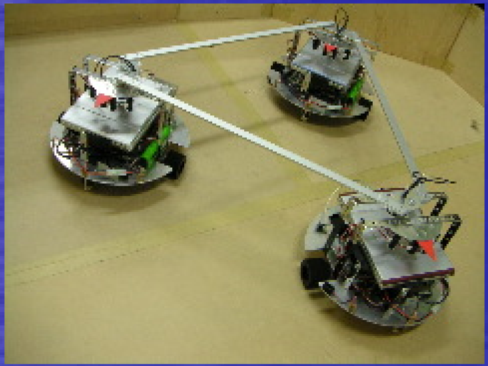
  Find $u_i(t)$ such that as $t \to \infty$ we have

$$x_1(t) = x_2(t) = \cdots = x_N(t),$$

Question: what is the minimum information needed for each agent?

Formation Control of Mobile Robots with Limited Sensor Information

- Potential applications:
  - Area coverage
  - Infrastructure security (line of sight)
  - Transportation
  - Target tracking
  - etc



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We assume at least relative error to at least one neighbor can be sensed:

$$x_i - x_j.$$

(Distance and direction)

We consider a controller of the following type:

$$u_i(t) = \sum_{j \in N_i} a_{ij}(x_j - x_i),$$

where $a_{ij} = a_{ji}$ are positive weights.

If we let $x = (x_1, \cdots, x_N)^T$, then

$$\dot{x} = -Lx,$$

where

$$L = D - A = diag(\sum_{j \neq 1} a_{1j}, \cdots, \sum_{j \neq N} a_{Nj}) - [a_{ij}].$$

Now define

$$\phi(x) = x^T L x = \frac{1}{2} \sum_{i=1}^{N} \sum_{j \in N_i} a_{ij} (x_j - x_i)^2.$$

The consensus problem is solved, namely as $t \to \infty$, $x_1(t) = \cdots = x_N(t)$, if and only if

$$\phi(x) = 0 \iff x_1 = x_2 = \cdots = x_N.$$

In fact, in this case

$$\lim_{t \to \infty} x_i(t) = \frac{1}{N} \sum_{i=1}^{N} x_i(0).$$

Connection to Graph

- We take graph as a collection of nodes (vertices), edges that connect the nodes, and weights on the edges, denoted by G=(V,E,A).

- We say a graph is *connected* if any two nodes are connected by edges.
- ***The consensus problem is solved if the associated graph is connected.***

Closing Remarks

- For nonlinear systems such as mobile robots, observability depends also on the control.
- For mobile robots using exteroceptive sensors, active sensing is especially important.
- Sensing and control should be treated in an integrated fashion.
- For multiple robot systems, active sensing also concerns the topological structure of the sensing data.

Main References

- Xiaoming Hu and Torvald Ersson, Active observers for nonlinear systems, *Automatica vol 40* ,2004.
- Simon Cedervall and Xiaoming Hu, Active Nonlinear Observers for Mobile Systems, Proc. of IEEE CDC 2004.
- David Anisi and Xiaoming Hu, Active observers for mobile robotic systems, proc. of MTNS 2006.
- Maja Karasalo, Linda-Maria Johansson, Xiaoming Hu and Karl-Henrik Johansson, Multi-Robot Terrain Servoing with Proximity Sensors, Proc. of IEEE ICRA, Barcelona, 2005.
- M. Egerstedt, X. Hu and A. Stotsky, Control of Mobile Platforms Using a Virtual Vehicle Approach, *IEEE Trans. Automat. Contr.*, vol. 46, 2001.
- R Olfati-Saber, R Murray, Consensus problems in networks of agents with switching topology and time-delays, *IEEE Trans. Automat. Contr.*, vol. 49, 2004.